A Digital Harmonic Rejection Mixer with Analog Harmonic Pre-suppression

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Abstract - A harmonic rejection mixer (HRM) employing analog harmonic pre-suppression and subsequent digital domain harmonic suppression is proposed and investigated. The analog harmonic pre-suppression relaxes the dynamic range requirements for the baseband building blocks of the receiver. As a result, their power consumption can be reduced. This allows the path count of the HRM to be increased in order to enhance its interference suppression capabilities. The theory is validated by simulations.

Keywords — Harmonic rejection mixer, wideband receiver, RF front end.

I. INTRODUCTION

Harmonic rejection mixers (HRMs) have been increasingly used in wideband receivers because they alleviate the harmonic mixing problem and thus considerably relax the preselection filtering requirements. An HRM is a complex mixer, consisting of several parallel operating conventional switching mixers, driven by a polyphase local oscillator (LO). The HRM output signal is a weighted sum of the signals downconverted by the elementary mixers. The HRM can be seen as a perfect multiplier, driven by an effective LO waveform, in which some harmonics are suppressed [1]. The basic HRM operation principle can be illustrated by Fig. 1.

Unfortunately, harmonic rejection ratios (HRRs) of HRMs are typically limited to 30-40 dB due to gain and phase mismatches. An efficient way for HRR improvement is to perform the suppression of harmonic interferers in a digital domain. In this case, the weighted summation is performed after digitization of the baseband (BB) signals, obtained from the outputs of the elementary mixers. The weighting factors are calculated adaptively using an appropriate algorithm. There is a formal analogy between HRMs and CDMA systems [2]. So, algorithms, developed for multiuser detection (MUD) can be readily adapted for using in digital HRMs.

In order to enhance interference suppression capabilities of digital HRMs, it is desirable to increase their path count, but the power consumption can become prohibitively high. This problem can be mitigated if a way for reducing the dissipation of the individual HRM paths is found.

The power consumption of an amplifier can be approximately expressed by the noise floor, multiplied by the dynamic range and divided by its efficiency. Therefore, the consumption of an amplifier can be reduced by narrowing the required dynamic range.

The dissipation of ADCs depends on their number of bits (NOB). For NOB > 9, one bit decrease of NOB can reduce the power consumption up to four times [3]. Therefore, the dissipation of ADCs can be also reduced by narrowing the required dynamic range, since the needed NOB is determined by the dynamic range of the converted signals.

If the mixers are capacitively loaded, the required dynamic range of the baseband blocks is determined exclusively by the levels of the interferers at the LO harmonics, since the interference at other frequencies is strongly attenuated.

In this paper, we propose an HRM combining partial analog harmonic pre-suppression (AHPS) and subsequent digital domain harmonic suppression, based on an MUD algorithm. AHPS relaxes the dynamic range requirements for the baseband blocks and thus reduces their power consumption. This allows the HRM path count for a given power budget to be increased. Alternatively, for a given path count, the applicability of HRMs in battery powered receivers can be improved.
The rest of the paper is organized as follows: Section II presents the block diagram and the model of the proposed AHPS-HRM. Section III deals with the design of the AHPS matrix. Section IV treats the algorithm for the subsequent digital harmonic suppression (HS). Section V describes some noise mechanisms, specific for the HRMs employing AHPS. Section VI presents the simulation results.

II. THE AHPS-HRM MODEL

The conceptual block diagram of an AHPS-HRM is presented in Fig. 2 (a). The \( M \)-phase LO produces \( M \) symmetrical pulse trains spaced at \( \pi / M \) from one another. Due to the half wave symmetry of the LO pulses, the even LO harmonics are naturally suppressed. \( M \) baseband signals \( b_1, b_2, \ldots, b_M \) are obtained at the output of the polyphase mixer. The AHPS circuit produces new \( M \) signals \( y_1, y_2, \ldots, y_M \), which are weighted sums of \( b_1, b_2, \ldots, b_M \). In \( y_1, y_2, \ldots, y_M \) the interferers at the LO harmonics are suppressed to a certain extent, which makes the dynamic range of \( y_1, y_2, \ldots, y_M \) narrower than that of \( b_1, b_2, \ldots, b_M \). The ultimate interference suppression is performed in a digital domain after the digitization of \( b_1, b_2, \ldots, b_M \).

In some implementations of AHPS-HRM the AHPS circuit can be a non-distinct block and is inseparable from the remaining circuits. An example for such HRMs can be seen in [1] and [4]. Although the proposed AHPS-HRM has some similarity with [1], (i) we propose an HRM with arbitrary path count and (ii) we use a MUD algorithm for the digital domain HS. The LMS interference cancellation, used in [Ru], has some shortcomings which are explained in [2].

The AHPS-HRM model can be illustrated by the diagram in Fig. 2 (b). The model of the \( M \)-phase mixer is similar to that in [2]. The signals, however, are real-valued for the sake of clarity. Only the I-channel is considered, but the processing in the Q-channel is essentially the same. The RF signals at the mixer input are given by

\[
r(t) = \sum_{k=1}^{K} \left[ x_{i,k}(t) \cos(2\pi k f_{LO} t) + x_{i,k}(t) \sin(2\pi k f_{LO} t) \right],
\]

where \( K \) is determined by the RF LPF parameters. For the I-channel the term \( x_{i,1}(t) \cos(2\pi f_{LO} t) \) is the desired signal, and the other terms, including \( x_{i,0}(t) \sin(2\pi f_{LO} t) \) are treated as interferers.

The pulses at the \( i \)-th LO output can be expressed as

\[
LO_i(t) = 2 \sum_{k=1}^{\infty} \alpha_k \cos[2\pi k f_{LO} t - k \theta_i],
\]

where \( \alpha_k \) is the relative level of the \( k \)-th LO harmonic, \( \theta_i = \theta_i + \phi_i = \pi / M + \phi_i \), and \( \theta_i \) and \( \phi_i \) are the nominal phase shift and the phase error of the \( i \)-th LO pulse train.

The \( n \)-th sample of the digitized baseband signal at the \( i \)-th polyphase mixer output is:

\[
b_i(n) = (1 + \gamma_i) \sum_{k=1}^{K} \left[ x_{i,k}(n) \cos(k\theta_i) + x_{i,k}(n) \sin(k\theta_i) \right],
\]

where \( \gamma_i \) is the relative gain error of the \( i \)-th HRM path.
(Further, the sample numbers will be omitted.)

In a matrix representation

\[ \mathbf{b} = \sum_{k=1}^{K} \mathbf{a}_k \mathbf{x}_k \mathbf{s}_{k,1} + \sum_{k=1}^{K} \mathbf{a}_k \mathbf{x}_k \mathbf{s}_{k,Q} = \mathbf{S} \text{diag}(\mathbf{a}) \mathbf{x}, \]  

(4)

where \( \mathbf{b} = [b_1, b_2, ..., b_M]^T \),

\[ \mathbf{x} = [x_1, x_2, ..., x_K, x_{1, Q}, x_{2, Q}, ..., x_{K, Q}]^T, \]

\( \mathbf{a} = [a_1, a_2, ..., a_M, a_1, a_2, ..., a_M] \) and

\[ \mathbf{S} = [s_{1,1}, s_{2,1}, ..., s_{1,M}, s_{1,Q}, s_{2,Q}, ..., s_{1,M}] \] is an \( M \times 2K \) matrix with columns

\[ \mathbf{s}_{k,1} = [(1 + \gamma_1) \cos(\theta_1), (1 + \gamma_2) \cos(\theta_2), ..., (1 + \gamma_M) \cos(\theta_M)]^T \]

and

\[ \mathbf{s}_{k,Q} = [(1 + \gamma_1) \sin(\theta_1), (1 + \gamma_2) \sin(\theta_2), ..., (1 + \gamma_M) \sin(\theta_M)]^T. \]

Formally, the vectors \( \mathbf{s}_{k,1} \) and \( \mathbf{s}_{k,Q} \) can be seen as signature vectors of the respective signals \( x_{k,1} \) and \( x_{k,Q} \) in a CDMA system. The objective is to separate the signal \( x_{k,1} \) with signature \( s_{1,1} \) from the interferers with signatures \( s_{k,1}, k \neq 1 \) and \( s_{k,Q}, k = 1, 2, ..., K \). (For the Q-channel the desired signal is \( x_{k,Q} \) with signature \( s_{1,Q} \).)

The introduction of AHPS can be expressed by an additional multiplication of \( \mathbf{b} \) by a matrix \( \mathbf{P} \), containing the weighting factors, ensuring the AHPS. The signal at the AHPS circuit output is:

\[ \mathbf{y} = \mathbf{P} \mathbf{b} = \mathbf{P} \mathbf{S} \text{diag}(\mathbf{a}) \mathbf{x} = \mathbf{S}_{\text{AHPS}} \text{diag}(\mathbf{a}) \mathbf{x}. \]  

(5)

So the columns of the matrix \( \mathbf{S}_{\text{AHPS}} = \mathbf{PS} \) can be seen as new signatures, obtained by the signals after the AHPS circuit.

We seek an appropriate matrix \( \mathbf{P} \) and an algorithm for processing of \( \mathbf{y} \) or an appropriate MUD, from the perspective of the CDMA analogy.

III. THE AHPS MATRIX

A. Problems related with AHPS

Two problems can arise when AHPS is introduced. The first one is the possible information loss. If the signal \( \mathbf{b} \) cannot be reconstructed on the basis of \( \mathbf{y} \), then it is not certain that a MUD operating on \( \mathbf{y} \) would give the same interference suppression, as an MUD operating on \( \mathbf{b} \). The second problem is the possible increase of the projection of the desired signal signature on the interference subspace. In order to explain the harmful consequences of such orthogonality impairment, we can express the MUD as a sum of two orthogonal components without loss of generality: \( \mathbf{h}_{\text{MUD}} = \mathbf{s}_{\text{AHPS}} \mathbf{x} + \mathbf{s}_{\text{AHPS}} \mathbf{x} = 0 \), where \( \mathbf{s}_{\text{AHPS}} \) is the desired signature. All possible MUDs, scaled to produce the same desired signal power at their outputs can be presented in this form. For complete interference suppression, \( \mathbf{h}_{\text{MUD}} \) should be orthogonal to the interference space. If \( \mathbf{s}_{\text{AHPS}} \) is not orthogonal to the interference space, then \( \mathbf{x} \neq 0 \) and the norm \( \| \mathbf{h}_{\text{MUD}} \| \) will increase.

The power of the desired signal at the MUD output is \( \mathbf{h}_{\text{MUD}}^T \mathbf{A}_{\text{MUD}}^2 \mathbf{s}_{\text{AHPS}} = A_{\text{MUD}}^2 \| \mathbf{h}_{\text{MUD}} \|^2 \), where \( A_{\text{MUD}} \) is the desired signal level. The component \( \mathbf{x} \) does not contribute to the output power of the desired signal. The noise power at the MUD output is \( \sum_{i=1}^{M} \sigma_i^2 h_i^2 = \sigma^2 \| \mathbf{h}_{\text{MUD}} \|^2 \), where \( \sigma^2 \) is the variance of the uncorrelated noise at the MUD input. If \( \mathbf{s}_{\text{AHPS}} \) is orthogonal to the interference space, then \( \mathbf{x} = 0 \) as depicted in Fig. 3 (a). Then \( \| \mathbf{h}_{\text{MUD}} \| \) will be minimal and the output noise level will be also minimal. If the orthogonality is impaired, \( \| \mathbf{h}_{\text{MUD}} \| \) will increase because of the presence of a nonzero \( \mathbf{x} \) as can be seen in Fig. 3 (b). Then, the noise level will also increase, whereas the output level of the desired signal will remain constant. Therefore, the output SNR will decrease. If the input SNR is low, the MUD seeking to maximize the signal-to-interference-plus-noise ratio (SINR), will decrease \( \| \mathbf{h}_{\text{MUD}} \| \), decreasing \( \mathbf{x} \), and the orthogonality between \( \mathbf{h}_{\text{MUD}} \) and the interference space will be impaired. Then the interference suppression will be compromised.

In sum, the AHPS matrix must meet the following requirements: (i) It should be non-singular with a relatively good condition number and (ii) It should preserve the orthogonality between the desired signature and the interference space as much as possible.
B. Basic idea of the AHPS matrix design

Each of the elements of \( y \) is a weighted sum of the polyphase mixer outputs with weighting factors taken from the respective row of \( P \). We select such values, which ensure some harmonic suppression. Then \( y \) can be considered as the output of \( M \) simultaneously operating HRMs. Further, we select the elements of \( P \) in such a way as the effective LO waveforms of these HRMs to be spaced at \( \pi/M \) from one another. Then the combination of the polyphase mixer and AHPS circuit will be equivalent to a new polyphase mixer. The harmonic interferers at its output will be attenuated to some extent. The output signals of this new polyphase mixer can be processed in the same way as the signals of a polyphase mixer without AHPS. However, the required dynamic range of the succeeding processing blocks will be considerably narrower. Alternative choices for the AHPS matrix entries are also possible but they will be examined in further works.

According to the considerations presented so far, a natural choice for the entries of \( P \) is:

\[
p_{i,j} = \cos \left( \frac{\pi}{M} (i - j + 1) \right), \ i, j = 1, 2, ..., M. \tag{6}
\]

However, it can be proven that such an AHPS matrix is singular [5]. The solution is to introduce some nonzero intentional “errors” in these values. Naturally, these errors will reduce the achievable HRRs of the AHPS circuit. Since the set of values is the same for all rows, it is sufficient to give the values only for the first row of \( P \):

\[
p_i = (1 + \varepsilon_i) \cos \left( \frac{\pi}{M} i \right) + \Delta_i \approx (1 + \delta_i) \cos \left( \frac{\pi}{M} i \right) + \Delta_i, \tag{7}
\]

where \( \Delta_i \) are the absolute intentional errors and \( \delta_i \) are the relative random errors. If the implementation is according [4], \( \delta_i \) are also the same for all rows.

It can also be found, that the condition number of \( P \) is equal to the largest of the all HRRs ensured by the AHPS matrix [5].

C. Interference signatures after AHPS

It was proven [5] that the new signatures, obtained by the interferers after AHPS, are approximately a sum of two terms. The first one is caused by the phase errors of the polyphase LO and the second one is caused by the errors in the entries of \( P \). The \( j \)-th element of the signature of the cosine interferer at \( k \)-th LO harmonic is given by:

\[
s_{AHPS,j,k}(j) = s_{\text{AHPS},j,k}(j) + s_{\Delta \text{AHPS},j,k}(j)
\]

\[
= kA(\phi,k) \cos \left( j \frac{\pi}{M} + \alpha(\phi,k) \right) + B(\delta,k) \cos \left( j \frac{\pi}{M} + \beta(\delta,k) \right)
\]

where \( \phi = [\phi_1, \phi_2, ..., \phi_M]^T \), \( \delta = [\delta_1, \delta_2, ..., \delta_M]^T \) and \( \Delta = [\Delta_1, \Delta_2, ..., \Delta_M]^T \) are the vectors containing the phase errors, random amplitude errors and intentional amplitude errors, respectively. The signatures for the sine interferers are similar to (8).

From (8) it can be seen that the terms, caused by the phase errors are half waves of sampled sinusoids having different phase angles, but with the same frequency as the signature of the desired signal. Therefore, in the general case the phase errors impair the orthogonality between the signatures. On the contrary, the terms caused by the amplitude errors are scaled and phase shifted replicas of the original signatures so they preserve the orthogonality.

As it was discussed above, the orthogonality impairment can result in SINR degradation. Therefore, it is desirable in the new signatures to dominate the terms caused by the amplitude errors.

D. The AHPS matrix design procedure

The starting point is an estimation of the minimum affordable variance of the phase errors for a given technology, power budget, etc. Then such an order of magnitude of the amplitude errors should be determined, which ensures that the terms caused by the amplitude errors will dominate the new signatures. This can be done using an equation giving the expected HRRs as a function of the phase and amplitude errors, for example the equation, derived in [6], or simply by performing some Monte Carlo simulations. The order of magnitude of the intentional amplitude errors should be greater than the determined order of magnitude and at least one order of magnitude greater than the minimum affordable random amplitude errors. At this point the HRRs of the AHPS block can also be estimated. Note that the HRRs will be practically determined by the values of the intentional errors.

Then an appropriate set of values for the first row of \( P \) ensuring the estimated HRRs should be found. It is advisable that all of the values be multiples of a given constant step, because in a practical implementation the entries of \( P \) will be realized by combinations of "unity" elements, e.g. unity resistors, unity transconductors, etc. If the AHPS weighting will be applied at RF, as it is in the HRM in [1] and [4], the number of unity elements should be minimal in order to reduce the circuit parasitics. The search of values for \( P \) can be formulated as an optimization...
Three examples for $M=8$ are given in Table I. The first one ensures a reduction of the NOB of the ADCs by 4 bits and can be used for phase errors and random amplitude errors of the order of tenths of a degree and 1%, respectively. Combination 2 allows somewhat larger errors, but the NOB reduction is by approximately 3 bits. Combination 3 gives the largest dynamic range reduction, but requires exceptional phase and amplitude accuracy. The number of unity elements is significantly larger than in 1 and 2, which can result in a lower corner frequency of the HRM.

<table>
<thead>
<tr>
<th>Table I. Sample combinations of AHPS weighting factors for $M=8$</th>
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<tbody>
<tr>
<td><strong>First row of $P$</strong></td>
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<tr>
<td>----------------------</td>
</tr>
<tr>
<td>$P_1 = [3, 3, 2, 1, 0, -1, -2, -3] 	imes 1/3$</td>
</tr>
<tr>
<td>$P_2 = [2, 2, 1, 1, 0, -1, -2, -2] 	imes 1/2$</td>
</tr>
<tr>
<td>$P_3 = [13, 11, 8, 4, -1, -1, -10, -12] 	imes 1/13$</td>
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</table>

IV. DIGITAL HS ALGORITHM

A. Choice of suitable MUD algorithm

A suitable MUD algorithm was found and adapted for digital domain HS. We wanted an MUD algorithm which is: near-far resistant; having low computational complexity; able to adapt to the signature mismatches without preliminary knowledge of the errors of the polyphase mixer and AHPS matrix; not requiring training sequences for its adaptation; robust to signature mismatches of a general kind (not related only to the RF propagation effects); not relying on a specific modulation type, encoding or patterns of the signals; not subspace-based (would be inefficient with regard to HRM hardware utilization).

It is well known that MUD algorithms are sensitive to the desired signature mismatch. A method for mismatch estimation was proposed in [2], but it cannot be adapted to HRM-AHPS since the mathematical expressions of the signatures become fairly complex in this case.

Numerous publications in the area of MUD and array processing were examined and the MUD presented in [7] have proven to be the most appropriate for our purposes. The selected MUD minimizes the largest value of the mean square error over all possible norm-bounded errors in the desired signature and data covariance matrix [7]. It is formally equivalent to the multiuser receivers using diagonal loading, but here the value of the diagonal loading factor is optimally chosen based on the level of uncertainty in the desired signal signature. In contrast to the MUD algorithm based on second order cone programming [8], the algorithm presented in [7] is computationally efficient and thus is suitable for online implementation. The MUD in [7] is in the form:

$$h = \left( \hat{\mathbf{R}} + \left( \gamma + \frac{\epsilon}{\tau} \right) \mathbf{I} \right)^{-1} \hat{s}_y,$$

where $\hat{\mathbf{R}}$ is the sample estimate of the covariance matrix of $\mathbf{y}$, $\hat{s}_y$ is the nominal desired user signature, $\gamma$ is a bound of the Frobenius norm of the error between the true covariance matrix of $\mathbf{y}$ and its sample estimate, and $\epsilon$ is a bound of the desired user signature error, $\mathbf{I}$ is an $M \times M$ identity matrix, and $\tau$ is the solution of the following equation:

$$\tau^2 = \left( \left( \hat{\mathbf{R}} + \left( \gamma + \frac{\epsilon}{\tau} \right) \mathbf{I} \right)^{-1} \hat{s}_y \right)^2.$$

The solution of (10) can be found in a computationally efficient manner as described in [7]. The simulations showed that in some scenarios $\gamma \neq 0$ results in considerable SINR degradation. This can be attributed to the fact that $\gamma$ introduces a non-adaptive component in the diagonal loading factor, thus worsening the adaptability of the detector. We found that the substitution $\gamma = 0$ leads to better results. The nonzero error of $\hat{\mathbf{R}}$ can be absorbed by $\epsilon$ if a little larger than the initially estimated value is selected. In addition, the simulations proved that a sample number of the order of 1000-2000 gives a sufficiently accurate covariance matrix estimate for a normal MUD operation.

B. Bound of the error norm of the desired user signature

We are seeking an $\epsilon$, such that $\epsilon < \| e(\phi, \delta) \|$, where $e(\phi, \delta) = \mathbf{s}_{AHPS1} - \hat{s}_{AHPS1}$ and $\mathbf{s}_{AHPS1}$ and $\hat{s}_{AHPS1}$ are the actual and the nominal desired signal signature after AHPS.
It was found in [5] that the signature error is approximately a sum of two independent terms: $e(\phi,0)$ and $e(0,\delta)$. The first one is caused by the phase errors and the second one by the amplitude errors.

It was found for the first term:

$$\|e(\phi,0)\| \leq \lambda_{\text{max}}(\mathbf{F}) \|\varphi\| \approx \frac{M}{2} \max \|\varphi\|,$$

where $\lambda_{\text{max}}(\mathbf{F})$ is the largest eigenvalue of the nominal (without random errors) AHPS matrix $\mathbf{F}$.

Since $\|\varphi\|^2$ has $\chi^2$ distribution, $\|\varphi\|$ is unbounded strictly speaking. However, for a given probability value $p$ holds

$$P[\|\varphi\|^2 < \sigma_\varphi f(p,M)] = p,$$

where the value of $f(p,M)$ can be found using a $\chi^2$-distribution table or the corresponding Matlab function.

Then, with probability $p$ holds:

$$\|e(\phi,0)\| \leq \sigma_\varphi \lambda_{\text{max}}(\mathbf{F}) f(p,M) \approx \sigma_\varphi \frac{M}{2} f(p,M)$$

Similarly, it was found:

$$\|e(0,\delta)\| \leq \sigma_\delta \lambda_{\text{max}}(\mathbf{F}) f(p,M) \approx \sigma_\delta \frac{M}{2} f(p,M).$$

Finally:

$$\varepsilon = \lambda_{\text{max}}(\mathbf{F}) f(p,M) \sqrt{\sigma_\varphi^2 + \sigma_\delta^2} \approx \frac{M}{2} f(p,M) \sqrt{\sigma_\varphi^2 + \sigma_\delta^2}$$

The derived bound $\varepsilon$ was confirmed by Matlab simulations.

V. NOISE MECHANISMS IN HRM-AHPS

The noise of MUD is strongly dependent on the figure "surplus energy" $\chi$ [9]. In the ideal case $\chi = 0$ and the output noise level of MUD rises when $\chi$ increases. The desired signal signature errors and the impairment of the orthogonality between the desired signature and the interference space result in larger $\chi$ values. Unfortunately, the estimation of $\chi$ is difficult. We performed some theoretical investigations and numerous simulations and found that $\chi$ is nearly zero if there are not strong interferers at a "problematic" set harmonics, i.e. pairs of harmonics of orders $k$ and $2M - k$. (See [10] for more details about "problematic" sets of harmonics.)

Three kinds of noise can be distinguished in an AHPS-HRM:

- Noise voltages obtaining signatures.
- Noise voltages without signatures getting at the input of the AHPS circuit.
- Noise voltages adding to the AHPS output signals.

Noise voltages obtaining signatures are the white noise components at the HRM input, centered around the LO harmonics. If there is no HRR maximization for problematic harmonic sets, the LO harmonics with orders $k \neq 2Mn \pm 1, n = 0,1,2,...$ are suppressed by the HRM (at least 20 dB under normal conditions), whereas the harmonics of orders $k = 2Mn \pm 1, n = 1,2,...$ are considerably attenuated due to their natural roll-off. Therefore, practically only the noise components centered around $f_{\text{LO}}$ (i.e. the noise components in the frequency band of the desired signal) are translated to baseband. So, the output SNR will be nearly the same as the input SNR.

When the HRR is maximized for a problematic set of harmonics, however, some of the harmonics outside this set can be raised instead of suppressed [10]. So, the noise components around their frequencies will be downconverted to baseband and a considerable SNR degradation can occur.

Noise voltages without signatures are the noise voltages caused by the individual elementary mixers and the possible buffers after them. Each of these noise voltages, passing through the AHPS circuit, obtains the signature of the desired signal, but shifted by a multiple of $\pi / M$. With respect to this kind of noise, it was proven that a SNR improvement of $10\lg M - 3$ dB is obtained at the MUD output when $\chi = 0$. When $\chi \neq 0$, an SNR degradation at the MUD output can occur only if the orthogonal component of the MUD has nonzero projections on some of the signatures obtained by the noise in the AHPS circuit. Therefore, if $\chi$ is large, a considerable SNR degradation is possible, but with a relatively small probability.

The third kind of noise includes the ADC quantization noise and the noise caused by the analog parts of the ADCs and the baseband amplifiers between AHPS and ADCs. As regards with this kind of noise, it was proven that a SNR alteration of $10\lg M - 3 - 10\lg (1 + \chi)$ dB is obtained at the MUD output. So, if the path count $M$ increases, the needed NOB of the ADCs decreases. This results in a decreased ADC dissipation that compensates the rise of the HRM consumption caused by the raised path count. On the other hand, large $\chi$ values cause a considerable SNR degradation. If an improperly designed AHPS matrix is used, $\chi$ will be high. The caused SNR degradation can be compensated mainly by reduction of the ADC quantization noise. Then, the necessary rise of the NOB and the corresponding rise of ADC power consumption can reduce

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the benefit of employing AHPS to zero. In conclusion, it is worth noting that there is a substantial difference between CDMA systems and HRMs with digital domain harmonic suppression with respect to the noise. In CDMA systems the noise does not obtain signatures. As a result, the MUD output noise level in a CDMA system is exclusively determined by the MUD surplus energy. In HRMs, large values can raise mainly the noise of the baseband buffers/amplifiers and the ADC quantization noise.

VI. SIMULATION RESULTS

HRMs with an AHPS matrix according to the first row of Table I were simulated. The expected reduction of the needed ADC NOB, caused by the introducing of AHPS is 4 bits for an interferer at $3f_{LO}$ (because $25.8/6 = 4$).

In Fig. 4 the medians of SINR for 100 realizations of HRMs with AHPS and without AHPS are presented. The SINR median is above 20 dB up to the point where SINR becomes limited by the ADC quantization noise. It can be seen that the ADC NOB should be raised from 12 to 16 when AHPS is not used for comparable results.

In Fig. 5 the noise limited output SINR of the HRM is presented for different kinds of noise. In Fig. 5 (a) it can be seen that the output SINR is nearly the same as the input SNR for the signed noise. In Fig. 5 (b) can be seen the SINR improvement of $10\log_{10}M - 3$ dB (in this case $10\log_{10}8 - 3 = 6$ dB), for the mixer noise as was predicted theoretically. The noise levels of each of the elementary mixers are equal and the SNR at the mixer output with the maximum desired signal level is 10 dB. The output SINR is about 16 dB, i.e., we have a 6 dB SINR improvement. Fig. 5 (c) and (d) show the 4 bit reduction of the needed ADC NOB when AHPS is used.

In order to verify the considerations made in Section III, AHPS-HRMs with AHPS matrices according to Table I were simulated for different phase and amplitude error variances. The results are presented in Fig. 6 and Fig. 7. In Fig. 6 (a) it can be seen that SINR rapidly degrades when the term caused by the phase errors becomes prevailing in the interferer signatures. In addition, in this case the variance of SINR is too large, or, in other words, the HRM performance becomes less predictable. This additionally lowers the guaranteed SINR value. A similar behavior can be seen in Fig. 7 when the random amplitude errors increase.

VII. CONCLUSIONS

Theoretically, the use of AHPS can degrade the performance of the HRM. However, the simulations showed that if an AHPS-HRM is designed according to the considerations presented in this paper, it performs as well as a comparable HRM without AHPS. The achieved narrowing of the required dynamic range of the baseband components agrees with the expectation that it will be nearly equal to the HRR of the AHPS circuit. Therefore, the proposed AHPS-HRM can achieve the pursued goals.

In order to simplify the hardware, an HRM configuration relying solely on the LO pulse symmetry for the suppression of the even harmonics was selected. In this case the HRRs for the even harmonics will be no more than 60 dB. Therefore, it is advisable to use an HRM configuration which ensures the possibility for a better suppression of the even harmonics at the cost of doubling the path count. This can increase the HRM power consumption, but it can be compensated partly or completely by the use of AHPS. Alternatively, a notch filter similar to the one presented in [11] can be placed before the HRM input for additional suppression of the even harmonics.
Fig. 5. Histograms of the AHPS-HRM output SINR, when it is limited by the noise. HRM with $M = 8$, $\sigma_\psi = 0.25^\circ$, $\sigma_\delta = 10^{-3}$, AHPS matrix according to Table I, Row 1 and an interferer at $3f_{LO}$ with level +80 dB above the desired signal. 
(a) There is only white noise at the HRM input (noise components obtaining signatures); (b) There is only mixer noise, 10 dB SNR at the mixer output having a maximal desired signal level; (c) There is only quantization noise, NOB=11, AHPS with HRR3 $\approx$ 26 dB; (d) There is only quantization noise, NOB=15, without AHPS.

Fig. 6. Simulation results for an AHPS-HRM with increasing phase errors: (a) Degradation of the median SINR when $\sigma_\psi$ changes from 0.02$^\circ$ to 2$^\circ$ for the AHPS matrices according to Table I. (b)-(d) SINR histograms when an AHPS matrix with HRR=25.8 dB is used and $\sigma_\psi = 0.02^\circ$, $\sigma_\psi = 0.2^\circ$ and $\sigma_\psi = 2^\circ$, respectively. There is a +80 dB interferer at $3f_{LO}$. The NOBs of the ADCs are selected for obtaining comparable results for small phase errors (correctly operating AHPS circuit).
Fig. 7. Simulation results for an AHPS-HRM with increasing random amplitude errors: (a) Degradation of the median SINR when $\sigma_{\text{AHPS}}$ changes from 0.1% to 32% for AHPS matrices according to Table I. (b)-(d) SINR histograms when an AHPS matrix with $\text{HRR}=25.8\, \text{dB}$ is used and $\sigma_{\text{AHPS}} = 1\%$, $\sigma_{\text{AHPS}} = 3\%$ and $\sigma_{\text{AHPS}} = 10\%$, respectively. There is a $+80\, \text{dB}$ interferer at $3f_{\text{LO}}$. NOBs of the ADCs are chosen for obtaining comparable results for small random amplitude errors (correctly operating AHPS circuit).

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[8] Shuguang Cui; Kisialiou, M.; Zhi-Quan Luo; Zhi Ding, "Robust blind multiuser detection against signature waveform mismatch based on second-order cone programming", Wireless Communications, IEEE Transactions on, vol. 4, no. 4, pp. 1285-1291, July 2005